BASUDEV GODABARI DEGREE COLLEGE , KESAIBAHAL

Department of Computer Science

"SELF STUDY MODULE"

Module Details :

- Class – 3rd Semester (2019-20) Admission Batch
- Subject Name : COMPUTER SCIENCE
- Paper Name : Discrete Mathematics

UNIT – 2 : STRUCTURE

2.1 Combinatory: Permutations,

2.2 Combinations, Pigeonhole principle

2.3 Recurrence Relation: Linear and Non-linear Recurrence Relations,

2.4 Solving Recurrence Relation using Generating Functions

You Can use the Following Learning Video link related to above topic :

https://youtu.be/NW-naslChdo

https://youtu.be/4V30R3I1vLI

https://youtu.be/3UeHl3UtmGI

https://youtu.be/5KUFK38m03I

You Can also use the following Books :

S.NO	Book Title	Author
1	Elements of Discrete Mathematics	C. L. Liu and D.P. Mohapatra
2	"Discrete Mathematical Structures with Applications to Computer Science	J. P Tremblay, R. Manohar

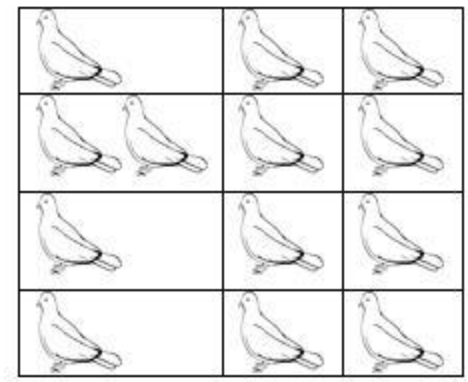
And also you can download any book in free by using the following website.

- https://www.pdfdrive.com/

Discrete Mathematics is a branch of mathematics involving discrete elements that uses algebra and arithmetic. It is increasingly being applied in the practical fields of mathematics and computer science. It is a very good tool for improving reasoning and problem-solving capabilities. This tutorial explains the fundamental concepts of Sets, Relations and Functions, Mathematical Logic, Group theory, Counting Theory, Probability, Mathematical Induction and Recurrence Relations, Graph Theory, Trees and Boolean Algebra.

Pigeon Hole Principal

Suppose that a flock of 20 pigeons flies into a set of 19 pigeonholes to roost. Because there are 20 pigeons but only 19 pigeonholes, a least one of these 19 pigeonholes must have at least two pigeons in it. To see why this is true, note that if each pigeonhole had at most one pigeon in it, at most 19 pigeons, one per hole, could be accommodated. This illustrates a general principle called the pigeonhole principle, which states that if there are more pigeons than pigeonholes, then there must be at least one pigeonhole with at least two pigeons in it.



Theorem –

I) If "A" is the average number of pigeons per hole, where A is not an integer then

- At least one pigeon hole contains **ceil[A]** (smallest integer greater than or equal to A) pigeons
- Remaining pigeon holes contains at most **floor[A]** (largest integer less than or equal to A) pigeons

Or

II) We can say as, if n + 1 objects are put into n boxes, then at least one box contains two or more objects.

The abstract formulation of the principle: Let X and Y be finite sets and let be a function.

- If X has more elements than Y, then f is not one-to-one.
- If X and Y have the same number of elements and f is onto, then f is one-to-one.
- If X and Y have the same number of elements and f is one-to-one, then f is onto.

Pigeonhole principle is one of the simplest but most useful ideas in mathematics. We will see more applications that proof of this theorem.

- **Example – 1:** If (Kn+1) pigeons are kept in n pigeon holes where K is a positive integer, what is the average no. of pigeons per pigeon hole?
  Solution: average number of pigeons per hole = (Kn+1)/n
  = K + 1/n
  Therefore there will be at least one pigeonhole which will contain at least (K+1) pigeons i.e., ceil[K +1/n] and remaining will contain at most K i.e., floor[k+1/n] pigeons.

i.e., the minimum number of pigeons required to ensure that at least one pigeon hole contains (K+1) pigeons is (Kn+1).

- **Example – 2:** A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color?
  Solution: Apply pigeonhole principle.
  No. of colors (pigeonholes) n = 3
  No. of marbles (pigeons) K+1 = 4
  Therefore the minimum no. of marbles required = Kn+1
  By simplifying we get Kn+1 = 10.
  Verification: ceil[Average] is [Kn+1/n] = 4
  [Kn+1/3] = 4
  Kn+1 = 10
  i.e., 3 red + 3 white + 3 blue + 1(red or white or blue) = 10

Pigeonhole principle strong form –
Theorem: Let $q_1, q_2, \ldots, q_n$ be positive integers.
If $q_1 + q_2 + \ldots + q_n - n + 1$ objects are put into n boxes, then either the 1st box contains at least q_1 objects, or the 2nd box contains at least q_2 objects, . . ., the nth box contains at least q_n objects.

Application of this theorem is more important, so let us see how we apply this theorem in problem solving.

- **Example – 1:** In a computer science department, a student club can be formed with either 10 members from first year or 8 members from second year or 6 from third year or 4 from final year. What is the minimum no. of students we have to choose randomly from department to ensure that a student club is formed?
  Solution: we can directly apply from the above formula where,
  $q_1 = 10$, $q_2 = 8$, $q_3 = 6$, $q_4 = 4$ and n=4
  Therefore the minimum number of students required to ensure department club to be formed is 10 + 8 + 6 + 4 – 4 + 1 = 25
- **Example – 2:** A box contains 6 red, 8 green, 10 blue, 12 yellow and 15 white balls. What is the minimum no. of balls we have to choose randomly from the box to ensure that we get 9 balls of same color?
  Solution: Here in this we cannot blindly apply pigeon principle. First we will see what happens if we apply above formula directly.
  From the above formula we have get answer 47 because 6 + 8 + 10 + 12 + 15- 5 + 1 = 47
  But it is not correct. In order to get the correct answer we need to include only blue, yellow and white balls because red and green balls are less than 9. But we are picking randomly so we include after we apply pigeon principle.
  i.e., 9 blue + 9 yellow + 9 white – 3 + 1 = 25
  Since we are picking randomly so we can get all the red and green balls before the above 25 balls. Therefore we add 6 red + 8 green + 25 = 39
  We can conclude that in order to pick 9 balls of same color randomly, one has to pick 39 balls from a box.

Recurrence Relation: Linear and Non-linear Recurrence Relations,

Definition

A recurrence relation is an equation that recursively defines a sequence where the next term is a function of the previous terms (Expressing F_n as some combination of F_i with $i<n$).

Example − Fibonacci series − $F_n=F_{n-1}+F_{n-2}$, Tower of Hanoi − $F_n=2F_{n-1}+1$

Linear Recurrence Relations

A linear recurrence equation of degree k or order k is a recurrence equation which is in the format $x_n=A_1x_{n-1}+A_2x_{n-1}+A_3x_{n-1}+\dots A_kx_{n-k}$ (A_n is a constant and $A_k\neq 0$) on a sequence of numbers as a first-degree polynomial.

These are some examples of linear recurrence equations –

Recurrence relations	Initial values	Solutions
$F_n = F_{n-1} + F_{n-2}$	$a_1 = a_2 = 1$	Fibonacci number
$F_n = F_{n-1} + F_{n-2}$	$a_1 = 1, a_2 = 3$	Lucas Number
$F_n = F_{n-2} + F_{n-3}$	$a_1 = a_2 = a_3 = 1$	Padovan sequence
$F_n = 2F_{n-1} + F_{n-2}$	$a_1 = 0, a_2 = 1$	Pell number

How to solve linear recurrence relation

Suppose, a two ordered linear recurrence relation is − $F_n=AF_{n-1}+BF_{n-2}$ where A and B are real numbers.

The characteristic equation for the above recurrence relation is −

$$x^2-Ax-B=0$$

Three cases may occur while finding the roots −

Case 1 − If this equation factors as $(x-x_1)(x-x_1)=0$ and it produces two distinct real roots x_1 and x_2, then $F_n=ax_1^n+bx_2^n$ is the solution. [Here, a and b are constants]

Case 2 − If this equation factors as $(x-x_1)^2=0$ and it produces single real root x_1, then $F_n=ax_1^n+bnx_1^n$ is the solution.

Case 3 − If the equation produces two distinct complex roots, x_1 and x_2 in polar form $x_1=r\angle\theta$ and $x_2=r\angle(-\theta)$, then $F_n=r^n(a\cos(n\theta)+b\sin(n\theta))$ is the solution.

Problem 1

Solve the recurrence relation $F_n=5F_{n-1}-6F_{n-2}$ where $F_0=1$ and $F_1=4$

Solution

The characteristic equation of the recurrence relation is −

$$x^2-5x+6=0,$$

So, $(x-3)(x-2)=0$

Hence, the roots are −

$x_1=3$ and $x_2=2$

The roots are real and distinct. So, this is in the form of case 1

Hence, the solution is −

$$F_n=ax_1^n+bx_2^n$$

Here, $F_n=a3^n+b2^n$ (As $x_1=3$ and $x_2=2$)

Therefore,

$1=F_0=a3^0+b2^0=a+b$
$4=F_1=a3^1+b2^1=3a+2b$
Solving these two equations, we get $a=2$ and $b=-1$

Hence, the final solution is −

$$F_n=2.3^n+(-1).2^n=2.3^n-2^n$$

Problem 2

Solve the recurrence relation − $F_n=10F_{n-1}-25F_{n-2}$ where $F_0=3$ and $F_1=17$

Solution

The characteristic equation of the recurrence relation is −

$$x^2-10x-25=0$$

So $(x-5)^2=0$
Hence, there is single real root $x_1=5$

As there is single real valued root, this is in the form of case 2

Hence, the solution is −

$F_n=ax_1^n+bnx_1^n$
$3=F_0=a.5^0+(b)(0.5)^0=a$
$17=F_1=a.5^1+b.1.5^1=5a+5b$
Solving these two equations, we get $a=3$ and $b=2/5$
Hence, the final solution is − $F_n=3.5^n+(2/5).n.2^n$

Problem 3

Solve the recurrence relation $F_n=2F_{n-1}-2F_{n-2}$ where $F_0=1$ and $F_1=3$

Solution

The characteristic equation of the recurrence relation is −

$$x^2-2x-2=0$$

Hence, the roots are −

$x_1=1+i$x1=1+i and $x_2=1-i$x2=1−i

In polar form,

$x_1=r\angle\theta$x1=r∠θ and $x_2=r\angle(-\theta)$,x2=r∠(−θ), where $r=\sqrt{2}$r=2 and $\theta=\frac{\pi}{4}$θ=π4

The roots are imaginary. So, this is in the form of case 3.

Hence, the solution is −

$F_n=(\sqrt{2})^n(a\cos(n.\sqcap/4)+b\sin(n.\sqcap/4))$Fn=(2)n(acos(n.⊓/4)+bsin(n.⊓/4))
$1=F_0=(\sqrt{2})^0(a\cos(0.\sqcap/4)+b\sin(0.\sqcap/4))=a$1=F0=(2)0(acos(0.⊓/4)+bsin(0.⊓/4))=a
$3=F_1=(\sqrt{2})^1(a\cos(1.\sqcap/4)+b\sin(1.\sqcap/4))=\sqrt{2}(a/\sqrt{2}+b/\sqrt{2})$3=F1=(2)1(acos(1.⊓/4)+bsin(1.⊓/4))=2(a/2+b/2)
Solving these two equations we get $a=1$a=1 and $b=2$b=2

Hence, the final solution is −

$F_n=(\sqrt{2})^n(\cos(n.\pi/4)+2\sin(n.\pi/4))$Fn=(2)n(cos(n.π/4)+2sin(n.π/4))

Non-Homogeneous Recurrence Relation and Particular Solutions

A recurrence relation is called non-homogeneous if it is in the form

$F_n=AF_{n-1}+BF_{n-2}+f(n)$Fn=AFn−1+BFn−2+f(n) where $f(n)\neq 0$f(n)≠0
Its associated homogeneous recurrence relation is $F_n=AF_{n-1}+BF_{n-2}$Fn=AFn–1+BFn−2
The solution (a_n)(an) of a non-homogeneous recurrence relation has two parts.
First part is the solution (a_h)(ah) of the associated homogeneous recurrence relation and the second part is the particular solution (a_t)(at).

$$a_n=a_h+a_t$$an=ah+at

Solution to the first part is done using the procedures discussed in the previous section.

To find the particular solution, we find an appropriate trial solution.

Let $f(n)=cx^n$f(n)=cxn ; let $x^2=Ax+B$x2=Ax+B be the characteristic equation of the associated homogeneous recurrence relation and let x_1x1 and x_2x2 be its roots.

- If $x\neq x_1$x≠x1 and $x\neq x_2$x≠x2, then $a_t=Ax^n$at=Axn
- If $x=x_1$x=x1, $x\neq x_2$x≠x2, then $a_t=Anx^n$at=Anxn
- If $x=x_1=x_2$x=x1=x2, then $a_t=An^2x^n$

Relation

Whenever sets are being discussed, the relationship between the elements of the sets is the next thing that comes up. **Relations** may exist between objects of the same set or between objects of two or more sets.

Definition and Properties

A binary relation R from set x to y (written as xRyxRy or $R(x,y)$R(x,y)) is a subset of the Cartesian product $x \times y$x×y. If the ordered pair of G is reversed, the relation also changes.
Generally an n-ary relation R between sets $A_1, \dots,$ and A_nA1,…, and An is a subset of the n-ary product $A_1 \times \dots \times A_n$A1×⋯×An. The minimum cardinality of a relation R is Zero and maximum is n^2n2 in this case.
A binary relation R on a single set A is a subset of $A \times A$A×A.

For two distinct sets, A and B, having cardinalities *m* and *n* respectively, the maximum cardinality of a relation R from A to B is *mn*.

Domain and Range

If there are two sets A and B, and relation R have order pair (x, y), then −

- The **domain** of R, Dom(R), is the set $\{x \mid (x,y) \in R \text{ for some } y \text{ in } B\}${x|(x,y)∈RforsomeyinB}
- The **range** of R, Ran(R), is the set $\{y \mid (x,y) \in R \text{ for some } x \text{ in } A\}${y|(x,y)∈RforsomexinA}

Examples

Let, $A=\{1,2,9\}$A={1,2,9} and $B=\{1,3,7\}$B={1,3,7}

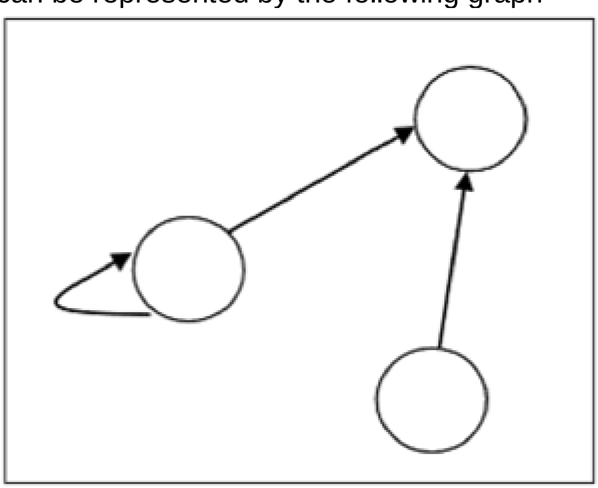
- Case 1 − If relation R is 'equal to' then $R=\{(1,1),(3,3)\}$R={(1,1),(3,3)}
  Dom(R) = $\{1,3\}, Ran(R)=\{1,3\}${1,3},Ran(R)={1,3}
- Case 2 − If relation R is 'less than' then $R=\{(1,3),(1,7),(2,3),(2,7)\}$R={(1,3),(1,7),(2,3),(2,7)}
  Dom(R) = $\{1,2\}, Ran(R)=\{3,7\}${1,2},Ran(R)={3,7}
- Case 3 − If relation R is 'greater than' then $R=\{(2,1),(9,1),(9,3),(9,7)\}$R={(2,1),(9,1),(9,3),(9,7)}
  Dom(R) = $\{2,9\}, Ran(R)=\{1,3,7\}${2,9},Ran(R)={1,3,7}

Representation of Relations using Graph

A relation can be represented using a directed graph.

The number of vertices in the graph is equal to the number of elements in the set from which the relation has been defined. For each ordered pair (x, y) in the relation R, there will be a directed edge from the vertex 'x' to vertex 'y'. If there is an ordered pair (x, x), there will be self- loop on vertex 'x'.

Suppose, there is a relation $R=\{(1,1),(1,2),(3,2)\}$R={(1,1),(1,2),(3,2)} on set $S=\{1,2,3\}$S={1,2,3}, it can be represented by the following graph −



Types of Relations

- The **Empty Relation** between sets X and Y, or on E, is the empty set $\emptyset$
- The **Full Relation** between sets X and Y is the set $X \times Y$
- The **Identity Relation** on set X is the set $\{(x,x) | x \in X\}$
- The Inverse Relation R' of a relation R is defined as − $R' = \{(b,a) | (a,b) \in R\}$
  Example − If $R = \{(1,2),(2,3)\}$ then R' will be $\{(2,1),(3,2)\}$
- A relation R on set A is called **Reflexive** if $\forall a \in A$ is related to a (aRa holds)

  Example − The relation $R = \{(a,a),(b,b)\}$ on set $X = \{a,b\}$ is reflexive.
- A relation R on set A is called **Irreflexive** if no $a \in A$ is related to a (aRa does not hold).
  Example − The relation $R = \{(a,b),(b,a)\}$ on set $X = \{a,b\}$ is irreflexive.
- A relation R on set A is called **Symmetric** if xRy implies yRx, $\forall x \in A$ and $\forall y \in A$.
  Example − The relation $R = \{(1,2),(2,1),(3,2),(2,3)\}$ on set $A = \{1,2,3\}$ is symmetric.
- A relation R on set A is called **Anti-Symmetric** if xRy and yRx implies $x = y \, \forall x \in A$ and $\forall y \in A$.
  Example − The relation $R = \{(x,y) \to N | x \leq y\}$ is anti-symmetric since $x \leq y$ and $y \leq x$ implies $x = y$.
- A relation R on set A is called **Transitive** if xRy and yRz implies $xRz, \forall x,y,z \in A$.
  Example − The relation $R = \{(1,2),(2,3),(1,3)\}$ on set $A = \{1,2,3\}$ is transitive.
- A relation is an **Equivalence Relation** if it is reflexive, symmetric, and transitive.

  Example − The relation $R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2),(1,3),(3,1)\}$ on set $A = \{1,2,3\}$ is an equivalence relation since it is reflexive, symmetric, and transitive.

Questions Bank

1. What Is Discrete Mathematics?

Answer :

Discrete Mathematics is a branch of mathematics involving discrete elements that uses algebra and arithmetic. It is increasingly being applied in the practical fields of mathematics and computer science. It is a very good tool for improving reasoning and problem-solving capabilities.

2. What Are The Categories Of Mathematics?

Answer :

Mathematics can be broadly classified into two categories −

Continuous Mathematics − It is based upon continuous number line or the real numbers. It is characterized by the fact that between any two numbers, there are almost always an infinite set of numbers. For example, a function in continuous mathematics can be plotted in a smooth curve without breaks.

Discrete Mathematics − It involves distinct values; i.e. between any two points, there are a countable number of points. For example, if we have a finite set of objects, the function can be defined as a list of ordered pairs having these objects, and can be presented as a complete list of those pairs.

3. What Is Sets In Discrete Mathematics?

Answer :

A set is an unordered collection of different elements. A set can be written explicitly by listing its elements using set bracket. If the order of the elements is changed or any element of a set is repeated, it does not make any changes in the set.

Some Example of Sets

- A set of all positive integers
- A set of all the planets in the solar system
- A set of all the states in India
- A set of all the lowercase letters of the alphabet

4. In How Many Ways Represent A Set?

Answer :

Sets can be represented in two ways −

Roster or Tabular Form: The set is represented by listing all the elements comprising it. The elements are enclosed within braces and separated by commas.

Example 1 − Set of vowels in English alphabet, A={a,e,i,o,u}A={a,e,i,o,u}

Example 2 − Set of odd numbers less than 10, B={1,3,5,7,9}

5. Explain Some Important Sets?

Answer :

- N − the set of all natural numbers = {1,2,3,4,.....}
- Z − the set of all integers = {.....,−3,−2,−1,0,1,2,3,.....}
- Z+ − the set of all positive integers
- Q − the set of all rational numbers
- R − the set of all real numbers
- W − the set of all whole numbers

6. What Is Cardinality Of A Set?

Answer :

Cardinality of a set S, denoted by |S|, is the number of elements of the set. The number is also referred as the cardinal number. If a set has an infinite number of elements, its cardinality is ∞.

Example − |{1,4,3,5}|=4,|{1,2,3,4,5,…}|=∞|

If there are two sets X and Y,

- |X|=|Y| denotes two sets X and Y having same cardinality. It occurs when the number of elements in X is exactly equal to the number of elements in Y. In this case, there exists a bijective function 'f' from X to Y.
- |X|≤|Y| denotes that set X's cardinality is less than or equal to set Y's cardinality. It occurs when number of elements in X is less than or equal to that of Y. Here, there exists an injective function 'f' from X to Y.
- |X|<|Y| denotes that set X's cardinality is less than set Y's cardinality. It occurs when number of elements in X is less than that of Y. Here, the function 'f' from X to Y is injective function but not bijective.
- If |X|≤|Y| and |X|≤|Y|then |X|=|Y|. The sets X and Y are commonly referred as equivalent sets.

7. What Are The Types Of Sets?

Answer :

Sets can be classified into many types. Some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Finite Set: A set which contains a definite number of elements is called a finite set.
Infinite Set: A set which contains infinite number of elements is called an infinite set.
Subset: A set X is a subset of set Y (Written as X⊆Y) if every element of X is an element of set Y.
Proper Subset: The term "proper subset" can be defined as "subset of but not equal to". A Set X is a proper subset of set Y (Written as X⊂YX⊂Y) if every element of X is an element of set Y and |X|<|Y|.
Universal Set: It is a collection of all elements in a particular context or application. All the sets in that context or application are essentially subsets of this universal set. Universal sets are represented as UU.

Empty Set or Null Set: An empty set contains no elements. It is denoted by ∅. As the number of elements in an empty set is finite, empty set is a finite set. The cardinality of empty set or null set is zero.
Singleton Set or Unit Set: Singleton set or unit set contains only one element. A singleton set is denoted by {s}.
Equal Set: If two sets contain the same elements they are said to be equal.
Equivalent Set: If the cardinalities of two sets are same, they are called equivalent sets.
Overlapping Set: Two sets that have at least one common element are called overlapping sets.
In case of overlapping sets −

- $n(A \cup B)=n(A)+n(B)-n(A \cap B)$
- $n(A \cup B)=n(A-B)+n(B-A)+n(A \cap B)$
- $n(A)=n(A-B)+n(A \cap B)$
- $n(B)=n(B-A)+n(A \cap B)$

Disjoint Set: Two sets A and B are called disjoint sets if they do not have even one element in common. Therefore, disjoint sets have the following properties −

- $n(A \cap B)=\emptyset$
- $n(A \cup B)=n(A)+n(B)$

8. What Is Set Operations?

- **Answer :**
- Set Operations include Set Union, Set Intersection, Set Difference, Complement of Set, and Cartesian Product.
- **Set Union:** The union of sets A and B (denoted by $A \cup B$) is the set of elements which are in A, in B, or in both A and B. Hence, $A \cup B=\{x|x \in A \text{ OR } x \in B\}$.
- **Set Intersection:** The intersection of sets A and B (denoted by $A \cap B$) is the set of elements which are in both A and B. Hence, $A \cap B=\{x|x \in A \text{ AND } x \in B\}$.
- Set Difference/ Relative Complement
- The set difference of sets A and B (denoted by A–B) is the set of elements which are only in A but not in B. Hence, $A-B=\{x|x \in A \text{ AND } x \notin B\}$.
- **Complement of a Set:** The complement of a set A (denoted by A′A′) is the set of elements which are not in set A. Hence, $A'=\{x|x \notin A\}$.
- More specifically, $A'=(U-A)$ where U is a universal set which contains all objects.

9. What Is Power Set?
Answer :
Power set of a set S is the set of all subsets of S including the empty set. The cardinality of a power set of a set S of cardinality n is 2n. Power set is denoted as P(S).

Example −For a set S={a,b,c,d} let us calculate the subsets −

- Subsets with 0 elements − {∅} (the empty set)
- Subsets with 1 element − {a},{b},{c},{d}
- Subsets with 2 elements − {a,b},{a,c},{a,d},{b,c},{b,d},{c,d}
- Subsets with 3 elements − {a,b,c},{a,b,d},{a,c,d},{b,c,d}
- Subsets with 4 elements − {a,b,c,d}

10. What Is Discrete Mathematics Relations?
Answer :
Whenever sets are being discussed, the relationship between the elements of the sets is the next thing that comes up. Relations may exist between objects of the same set or between objects of two or more sets.

11. What Is Discrete Mathematics Functions?
Answer :
A Function assigns to each element of a set, exactly one element of a related set. Functions find their application in various fields like representation of the computational complexity of algorithms, counting objects, study of

sequences and strings, to name a few. The third and final chapter of this part highlights the important aspects of functions.

12. What Is Composition Of Functions?

Answer :

Two functions f:A→Bf:A→B and g:B→Cg:B→C can be composed to give a composition gof. This is a function from A to C defined by (gof)(x)=g(f(x))

13. What Are Contradictions?

Answer :

A Contradiction is a formula which is always false for every value of its propositional variables.

Example − Prove (A∨B)∧[(¬A)∧(¬B)] is a contradiction

14. True FALSE The Pigeonhole Principle tells us that if we have n + 1 pigeons and n holes, since n + 1 > n, each box will have at least one pigeon.

Solution: One hole could have all n + 1 pigeons.

5. True FALSE The Pigeonhole Principle tells us that with n pigeons and k holes each hole can have at most n/k pigeons.

Solution: There exists one box with at least that many, but it could contain more.

6. Show that in a 8×8 grid, it is impossible to place 9 rooks so that they all don't threaten each other.

Solution: By Pigeonhole, there exists one row with at least two rooks, so they must threaten each other.

7. The population of the US is 300 million. Every person has written somewhere between 0 and 10 million lines of code. What's the maximum number of people that we can say must have written the same number of lines of code?

Solution: There are 10 · 106 + 1 different number of lines of code you can write. So, there exists a number of line of codes with at least d300 · 106/(106 + 1)e = 30 people.

8. Three people are running for student government. There are 202 people who vote. What is the minimum number of votes needed for someone to win the election?

Solution: By pigeonhole, there exists a person who has gotten at least d202/3e = 68 votes. So, someone could win with a 67 − 67 − 68 split.

9. There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, what is the minimum number of different rooms that will be needed?

Solution: There exists a time period will have at least d677/38e = 18 classes during it. So 18 different rooms will be needed.